16. Parametrize simply using x and y. The relevant upper bounds are simply:

$$\begin{aligned} 2x + 6(0) + 3(0) &= 12 \\ x &= 6 \\ 2x + 6y + 3(0) &= 12 \\ y &= 2 - \frac{x}{3} \end{aligned}$$

The parametrized surface can be written as $r(x, y) = \left\langle x,\, y,\, 4 - \frac{2}{3}x - 2y \right\rangle$, so the cross product is

$$\begin{aligned} r_{x(x,y)} &= \left\langle 1,\, 0,\, -\frac{2}{3} \right\rangle \\ r_{y(x,y)} &= \langle 0,\, 1,\, -2 \rangle \\ r_{x(x,y)} \times r_{y(x,y)} &= \left\langle \frac{2}{3},\, 2,\, 1 \right\rangle. \end{aligned}$$

This normal indeed has positive components. F simply needs the substitution for z, resulting in

$$\begin{aligned} F(x, y) &= \left\langle xy,\, y^2,\, y\left(4 - \frac{2}{3}x - 2y\right) \right\rangle \\ &= \left\langle xy,\, y^2,\, 4y - \frac{2}{3}xy - 2y^2 \right\rangle. \end{aligned}$$

The flux is then

$$\begin{aligned} \int_0^6 \int_0^{2-\frac{x}{3}} \left\langle xy,\, y^2,\, 4y - \frac{2}{3}xy - 2y^2 \right\rangle \cdot \left\langle \frac{2}{3},\, 2,\, 1 \right\rangle \mathrm{d}y\, \mathrm{d}x &= \int_0^6 \int_0^{2-\frac{x}{3}} \frac{2}{3}xy + 2y^2 + 4y - \frac{2}{3}xy - 2y^2 \,\mathrm{d}y\, \mathrm{d}x \\ &= \int_0^6 \int_0^{2-\frac{x}{3}} 4y \,\mathrm{d}y\, \mathrm{d}x \\ &= \int_0^6 2\left(2 - \frac{x}{3}\right)^2 \mathrm{d}x \\ &= 2\int_0^6 4 + \frac{x^2}{9} - 4\frac{x}{3} \,\mathrm{d}x \\ &= 2\left(4x + \frac{x^3}{27} - 2\frac{x^2}{3}\right)\Bigg|_0^6 \\ &= 2\left(4(6) + \frac{6^3}{27} - 2\frac{(6)^2}{3}\right) \\ &= 16. \end{aligned}$$

20. The cross product of the two partials:

$$\begin{aligned} r_u &= \langle \cos u \cos v, \cos u \sin v, -\sin u \rangle \\ r_v &= \langle -\sin u \sin v, \sin u \cos v,\, 0 \rangle \\ r_u \times r_v &= \langle \sin^2 u \cos v, \sin^2 u \sin v, \cos u \sin u \rangle \end{aligned}$$

which indeed points outward. The vector field after substitutions:

$$F(u, v) = \langle 4 \sin u \cos v \cos u,\, 4 \sin u \sin v \cos u,\, 4 \cos^2 u \rangle$$

And the integrand simplifies nicely:

$$
\begin{aligned}
&F(u,v)\cdot(r_u\times r_v)\\
&= 4\sin u\cos u\sin^2 u\cos^2 v + 4\sin u\cos u\sin^2 u\sin^2 v + 4\cos^2 u\times\cos u\sin u = 4\sin u\cos u\sin^2 u + 4\cos^2 u\times\cos u\sin u\\
&= 4\cos u\sin u\\
&= 2\sin(2u)
\end{aligned}
$$

whose antiderivative is just $-\cos(2u)$. Integrating:

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}}\int_0^{2\pi} 2\sin(2u)\,\mathrm{d}v\,\mathrm{d}u &= 2\pi(-\cos(2u))_0^{\frac{\pi}{2}}\\
&= 2\pi(-\cos(2u))_0^{\frac{\pi}{2}}\\
&= 4\pi.
\end{aligned}
$$

22. The xy-plane bound is a unit circle; we can parametrize using polar coordinates with $x = r\cos\theta, y = r\sin\theta, z = 0$ with $r\in[0,1], \theta\in[0,2\pi]$. The function then becomes

$$F(r,\theta) = \langle r\cos\theta,\ 0,\ r^2\cos^2\theta\rangle.$$

Finding the cross of the partials:

$$
\begin{aligned}
r_r &= \langle\cos\theta,\sin\theta,\ 0\rangle\\
r_\theta &= \langle -r\sin\theta,\ r\cos\theta,\ 0\rangle\\
r_r\times r_\theta &= \langle 0,\ 0,\ r\rangle.
\end{aligned}
$$

However, this puts the normal vector pointing upwards (into the solid), so we need to negate it to get the correct orientation. The integral is then

$$
\begin{aligned}
-\int_0^{2\pi}\int_0^1 F(r,\theta)\cdot(r_r\times r_\theta)\,\mathrm{d}r\,\mathrm{d}\theta &= -\int_0^{2\pi}\int_0^1 r^3\cos^2\theta\,\mathrm{d}r\,\mathrm{d}\theta\\
&= -\frac{1}{4}\int_0^{2\pi}\cos^2\theta\,\mathrm{d}\theta\\
&= -\frac{\pi}{4}.
\end{aligned}
$$

For the paraboloid, we use cylindrical coordinates and parametrize as $x = r\cos\theta, y = r\sin\theta, z = 1 - r^2$ with the same bounds on r and θ. Then,

$$F(r,\theta) = \langle r\cos\theta,\ 1-r^2,\ r^2\cos^2\theta\rangle.$$

The cross of the partials:

$$
\begin{aligned}
r_r &= \langle\cos\theta,\sin\theta,-2r\rangle\\
r_\theta &= \langle -r\sin\theta,\ r\cos\theta,\ 0\rangle\\
r_r\times r_\theta &= \langle 2r^2\cos\theta,\ 2r^2\sin\theta,\ r\rangle.
\end{aligned}
$$

This does correctly point outwards. Integrating:

$$\begin{aligned}&\int_0^{2\pi}\int_0^1 r\cos\theta\times 2r^2\cos\theta+(1-r^2)2r^2\sin\theta+r^2\cos^2\theta\times r\,\mathrm{d}r\,\mathrm{d}\theta\\&=\int_0^1\int_0^{2\pi}2r^3\cos^2\theta+2r^2\sin\theta-2r^4\sin\theta+r^3\cos^2\theta\,\mathrm{d}\theta\,\mathrm{d}r.\end{aligned}$$

Due to symmetry, the $\sin(\theta)$ terms go to zero from $0\to 2\pi$ so we can simply remove those:

$$\begin{aligned}\ldots&=\int_0^1\int_0^{2\pi}3r^3\cos^2\theta\,\mathrm{d}\theta\,\mathrm{d}r\\&=3\pi\int_0^1 r^3\,\mathrm{d}r\\&=\frac{3}{4}\pi.\end{aligned}$$

Adding, we obtain a total flux of $\frac{3}{4}\pi-\frac{\pi}{4}=\frac{\pi}{2}$.

28. Use cylindrical coordinates, fixing $r=4$, with $\theta\in[0,2\pi]$ and $z\in[-2,2]$, so $r(\theta,z)=\langle 4\cos\theta,\ 4\sin\theta,\ z\rangle$. Then, the cross:

$$\begin{aligned}r_\theta&=\langle -4\sin\theta,\ 4\cos\theta,\ 0\rangle\\r_z&=\langle 0,\ 0,\ 1\rangle\\r_\theta\times r_z&=\langle 4\cos\theta,\ 4\sin\theta,\ 0\rangle.\end{aligned}$$

Then,

$$F(\theta,z)=\langle 3,-7,\ z\rangle.$$

Integrate:

$$\begin{aligned}\int_0^{2\pi}\int_{-2}^2 12\cos\theta-28\sin\theta\,\mathrm{d}z\,\mathrm{d}\theta\\&=\int_0^{2\pi}48\cos\theta-112\sin\theta\,\mathrm{d}\theta\\&=0.\end{aligned}$$

by symmetry of trig functions over $[0,2\pi]$.

30a. Parametrize using cylindrical coordinates: $s(r,\theta)=\left\langle r\cos\theta,\ r\sin\theta,\ \sqrt{9-r^2}\right\rangle$. Plug into the vector field:

$$F(r,\theta)=\left\langle -2r\cos\theta,\ 0,\ 2\sqrt{9-r^2}+1\right\rangle.$$

The cross of the partials:

$$\begin{aligned}s_r&=\left\langle\cos\theta,\sin\theta,-\frac{r}{\sqrt{9-r^2}}\right\rangle\\s_\theta&=\langle -r\sin\theta,\ r\cos\theta,\ 0\rangle\\s_r\times s_\theta&=\left\langle r^2\cos\theta\frac{1}{\sqrt{9-r^2}},\ r^2\sin\theta\frac{1}{\sqrt{9-r^2}},\ r\right\rangle.\end{aligned}$$

Integrate:

$$\int_0^{2\pi}\int_0^3 \frac{-2r^3\cos^2\theta}{\sqrt{9-r^2}} + 2r\sqrt{9-r^2} + r\,\mathrm{d}r\,\mathrm{d}\theta.$$

For the inner integral, use $u = 9 - r^2, \mathrm{d}u = -2r\,\mathrm{d}r$:

$$\begin{aligned}\int_0^3 \frac{-2r^3\cos^2\theta}{\sqrt{9-r^2}} + 2r\sqrt{9-r^2} + r\,\mathrm{d}r &= \int_9^0 \frac{(9-u)\cos^2\theta}{\sqrt{u}} - \sqrt{u} - \frac{1}{2}\,\mathrm{d}u \\ &= \int_9^0 9u^{-\frac{1}{2}}\cos^2\theta - u^{\frac{1}{2}}\cos^2(\theta) - u^{\frac{1}{2}} - \frac{1}{2}\,\mathrm{d}u \\ &= -36\cos^2(\theta) + \frac{45}{2}.\end{aligned}$$

Finally:

$$\int_0^{2\pi} -36\cos^2(\theta) + \frac{45}{2}\,\mathrm{d}\theta = 9\pi.$$

The answer is thus 9π meters cubed.

b. Simply multiply the above answer to obtain 9,540 kg of fluid.